JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 28, No. 1, January–February 2005

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Output Feedback–Based Discrete-Time Sliding-Mode Controller Design for Model Aircraft

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Nomenclature

a_N = normal acceleration
u = forward velocity of aircraft
Z_α/M_α = force/moment derivative w. r. t. α
α = angle of attack
δ_e/δ_t = elevator/thrust deflection
δ_{ec}/δ_{tc} = control input to elevator/thrust deflector
θ/q = pitch angle/rate

I. Introduction

MODEL aircraft provide a low-cost test bed for the design and validation of control algorithms. The control system should improve the transient response and have good disturbance-rejection properties. The stability derivatives of the aircraft vary due to in-flight velocity fluctuations and the controller should be invariant to such perturbations. Weight and power requirements of the aircraft payload restrict the number of onboard sensors and so a limited number of output measurements can be used for feedback.

Variable structure control (VSC) systems have been widely discussed[1] and used in many fields of application. Of particular interest in VSC is the "sliding mode," which is obtained by designing a control law to drive the system states to reach and remain on the intersection of a set of prescribed switching surfaces. These modes are used to maintain the given constraints with high accuracy and are robust to both external disturbances and internal parameter perturbations. The use of full state information for sliding-mode design yields closed-loop systems that exhibit robustness to parameter variations. In cases where it is not possible to measure all states for feedback, the output vector may be used to define the switching function.[2–6] The lack of complete state information may be overcome by the use of a fixed-order dynamic compensator. The switching function is then defined using both the outputs and the compensator states[7,8] or using only the compensator states.[9] The first method requires nonsingularity of the product of the matrix that characterizes the switching surface with the output and input matrices of the system, which is not satisfied in many practical cases. The second method overcomes this difficulty but does not possess the desired degree of robustness to plant parameter perturbations. Also, the above techniques are applicable only to continuous-time design.

A new technique is developed for designing a discrete-time sliding-mode controller for irregular plants using the states of a compensator and part of the output vector to characterize the sliding manifold. Transformation of the plant model to regular form[10] is not carried out in this design procedure. A new set of conditions are imposed on the partitioned state and output matrices of the plant model, which may be satisfied by reorganizing the state vector. The control law for discrete-domain implementation is formulated and the reduced-order system is designed using the equivalent control approach.

II. System Model with Fixed-Order Compensator

Consider the discrete-time model of a linear time-invariant system:

$$x(k+1) = Ax(k) + Bu(k), \qquad y(k) = Cx(k) \tag{1}$$

where $x \epsilon \Re^n$, $u \epsilon \Re^m$, and $y \epsilon \Re^r$ are the state, input, and output vectors, respectively, $r > m$. The system matrices A, B, and C are of appropriate dimension. A fixed-order dynamic compensator of the form

$$z(k+1) = Pz(k) + Ny(k) \tag{2}$$

is considered, where $z \epsilon \Re^m$. The system model is partitioned as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{3}$$

where $x_1 \epsilon \Re^{n-m}$, $x_2 \epsilon \Re^m$, $y_1 \epsilon \Re^{r-m}$, and $y_2 \epsilon \Re^m$ and the following conditions, which may be satisfied by a simple reorganization of the state vector, hold:

$$\det(C_{22}) \neq 0 \tag{4}$$

$$C_{21} = 0 \tag{5}$$

$$A_{21} = 0 \tag{6}$$

In contrast to previous approaches,[7,8] where all of the outputs were used to characterize the sliding surface, we utilize a part y_2 of the output vector along with compensator states z to define the sliding surface $\sigma \epsilon \Re^m$. This alleviates the problems associated with singularity of the matrix product SCB:

$$\sigma(k) = S \begin{bmatrix} y_2(k) \\ z(k) \end{bmatrix} = \begin{bmatrix} S_p & S_c \end{bmatrix} \begin{bmatrix} y_2(k) \\ z(k) \end{bmatrix} \tag{7}$$

where $S_p \epsilon \Re^{m \times m}$ and $S_c \epsilon \Re^{m \times m}$ are invertible matrices. The discontinuous control is a function of the entire output y and compensator

Received 9 April 2004; revision received 14 July 2004; accepted for publication 30 July 2004.

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states z and is given by

$$u_i = \begin{cases} u_i^+(y,z) & \text{if} \quad \sigma_i > 0 \\ u_i^-(y,z) & \text{if} \quad \sigma_i < 0 \end{cases} \qquad i = 1, 2, \ldots, m \tag{8}$$

where σ_i's are elements of the m-dimensional switching vector $\sigma(k) = [\sigma_1 \;\; \sigma_2 \ldots \sigma_m]^T$.

III. Control Law and Reduced-Order System Design

The control law is derived using the equivalent-control approach.[11,12] To obtain a discrete VSC system with stable quasi-sliding mode,[13] the following condition should be satisfied:

$$\triangle\sigma(k+1) = \sigma(k+1) - \sigma(k) = 0 \tag{9}$$

Using Eqs. (3) and (7) along with conditions (4) and (5) in Eq. (9), we obtain the equivalent control

$$u_{\text{eq}}(k) = Gy(k) + Hz(k) \tag{10}$$

where $G \epsilon \Re^{m \times r}$ and $H \epsilon \Re^{m \times m}$ are the feedback matrices given by

$$G = -(S_p C_{22} B_2)^{-1} \left[S_c N_1 \quad \left(S_p C_{22} A_{22} C_{22}^{-1} + S_c N_2 - S_p \right) \right]$$

$$H = -(S_p C_{22} B_2)^{-1} S_c (P - I_m) \tag{11}$$

The compensator matrix N is partitioned as $N = [N_1 \;\; N_2]$ so that $N_1 \epsilon \Re^{m \times (r-m)}$ and $N_2 \epsilon \Re^{m \times m}$. Note that the matrix product $S_p C_{22} B_2$ is invertible and so the problems associated with nonsingularity of SCB are overcome.

The system on the sliding mode is governed by

$$\sigma(k) = 0 \Rightarrow y_2(k) = -S_p^{-1} S_c z(k)$$

Using this, we get the reduced state equations

$$x_1(k+1) = A_{11} x_1(k) - A_{12}(S_p C_{22})^{-1} S_c z(k) + B_1 u(k) \tag{12}$$

$$z(k+1) = N_1 C_{11} x_1(k) + \left(P - N_1 C_{12} (S_p C_{22})^{-1} S_c - N_2 S_p^{-1} S_c \right) z(k) \tag{13}$$

with the equivalent control

$$u_{\text{eq}}(k) = G_1 C_{11} x_1(k) + \left(H - \left(G_1 C_{12} C_{22}^{-1} + G_2 \right) S_p^{-1} S_c \right) z(k) \tag{14}$$

where the gain matrix G is partitioned as $G = [G_1 \;\; G_2]$ so that $G_1 \epsilon \Re^{m \times (r-m)}$ and $G_2 \epsilon \Re^{m \times m}$. We can then write the augmented reduced-order system in the general equivalent static output feedback form

$$x_a(k+1) = (A_a + B_a K_a C_a) x_a(k) \tag{15}$$

where the augmented state vector $x_a(k) = [x_1(k)^T \;\; z(k)^T]^T \epsilon \Re^n$ and

$$A_a = \begin{bmatrix} A_{11} & -A_{12}(S_p C_{22})^{-1} S_c \\ 0 & 0 \end{bmatrix}, \qquad B_a = \begin{bmatrix} B_1 & 0 \\ 0 & I \end{bmatrix}$$

$$C_a = \begin{bmatrix} C_{11} & -C_{12}(S_p C_{22})^{-1} S_c \\ 0 & I \end{bmatrix}$$

$$K_a = \begin{bmatrix} G_1 & \left(H - G_2 S_p^{-1} S_c \right) \\ N_1 & \left(P - N_2 S_p^{-1} S_c \right) \end{bmatrix} \tag{16}$$

Using a multivariable feedback design technique[14] for the triplet (A_a, B_a, C_a), we evaluate K_a and then partition it to get the parameter matrices of the controller.

With the equivalent control u_{eq} as derived in Eq. (10), a switching controller is formulated. To reduce chatter, we use a smoothing function that not only satisfies the reaching condition[15] but also produces an effect resembling that of a sliding sector in discrete VSC:

$$u = u_{\text{eq}} - \alpha (S_p C_{22} B_2)^{-1} \tanh(\sigma/\varepsilon), \qquad \varepsilon > 0 \tag{17}$$

The control inputs u_i^+ and u_i^- in Eq. (8) correspond to the values of u in Eq. (17) with $\sigma > 0$ and $\sigma < 0$, respectively.

IV. Numerical Example

A model aircraft has been developed by the Department of Aerospace Engineering, Indian Institute of Science, to study the feasibility of thrust vectoring in aircraft. The aircraft has a delta-wing configuration without a horizontal tail or canard and propulsion is in the form of a ducted fan driven by a piston engine. Thrust vectoring is incorporated by using tilting flaps in the jet at the end of the duct. The airplane is built using low-density materials to reduce weight and is remotely controlled. Stability derivatives of the aircraft were evaluated by wind tunnel testing. The actuators are modeled as first-order systems with a time constant of 50 ms. The discrete-time model for pitch dynamics of the aircraft at a nominal wind speed of 25 m/s is obtained with a sampling time of 20 ms:

$$A = \begin{bmatrix} 0.992 & 0.018 & -0.001 & 0.000 & -0.014 & -0.022 \\ -0.629 & 0.939 & 0.000 & 0.000 & -0.466 & -0.493 \\ 0.052 & -0.001 & 0.997 & -0.196 & -0.007 & -0.004 \\ -0.006 & 0.019 & 0.000 & 1.000 & -0.005 & -0.005 \\ 0 & 0 & 0 & 0 & 0.670 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.670 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.003 & -0.004 \\ -0.101 & -0.107 \\ -0.002 & -0.001 \\ -0.001 & -0.001 \\ 0.330 & 0 \\ 0 & 0.330 \end{bmatrix}$$

$$C = \begin{bmatrix} -9.499 & -0.570 & -0.830 & 0 & -19.930 & -32.905 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The state vector is given by $x = [\alpha \;\; q \;\; u \;\; \theta \;\; \delta_e \;\; \delta_t]^T$, the outputs are given by $y = [y_1^T \;:\; y_2^T] = [a_N \;\; q \;:\; \delta_e \;\; \delta_t]^T$, and the control inputs are given by $u = [\delta_{ec} \;\; \delta_{tc}]^T$.

Though the longitudinal dynamics of the aircraft are stable, large deviations from the calculated aerodynamic parameters are expected during flight. The small size of the airplane also makes it highly susceptible to wind disturbances, such as gusts. A controller is needed for flight stabilization under all operating conditions. VSC is chosen because it provides stability and desired performance even with large parameter perturbations in the plant.

In our case, we can partition the system matrices as in Eq. (3) without any reorganization of states so that $C_{22} \epsilon \Re^{2 \times 2}$ is invertible, $C_{21} = 0$, and $A_{21} = 0$. A second-order compensator of the form in Eq. (2) is used. Proceeding by manual tuning, we use

$$S_p = \begin{bmatrix} 0.40 & -0.32 \\ -0.30 & 0.40 \end{bmatrix}, \qquad S_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

to form the augmented matrices A_a, B_a, and C_a as in Eqs. (16). Output feedback–based eigenstructure assignment is used to find the pseudofeedback gain K_a. The aim has been to decouple the short-period and phugoid modes of the aircraft when the system is in the sliding mode. We get

$$K_a = \begin{bmatrix} -0.047 & 0.104 & 13.294 & 14.383 \\ -0.005 & 0.627 & 49.569 & 51.637 \\ -0.037 & -0.066 & 11.055 & 11.598 \\ -0.012 & 0.000 & 3.473 & 4.361 \end{bmatrix}$$

Choose

$$P = \begin{bmatrix} 0.65 & 0 \\ 0 & 0.60 \end{bmatrix}, \qquad H = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

On partitioning K_a appropriately and using the chosen values of P and H, we get

$$G = \begin{bmatrix} -0.047 & 0.104 & -0.993 & -1.491 \\ -0.005 & 0.627 & -4.327 & -4.785 \end{bmatrix}$$

$$N = \begin{bmatrix} -0.037 & -0.066 & -0.683 & -1.309 \\ 0.012 & 0.000 & -0.261 & -0.393 \end{bmatrix}$$

The system response to an initial value of $\alpha = 5$ deg is shown (Fig. 1). All the states governing longitudinal dynamics undergo

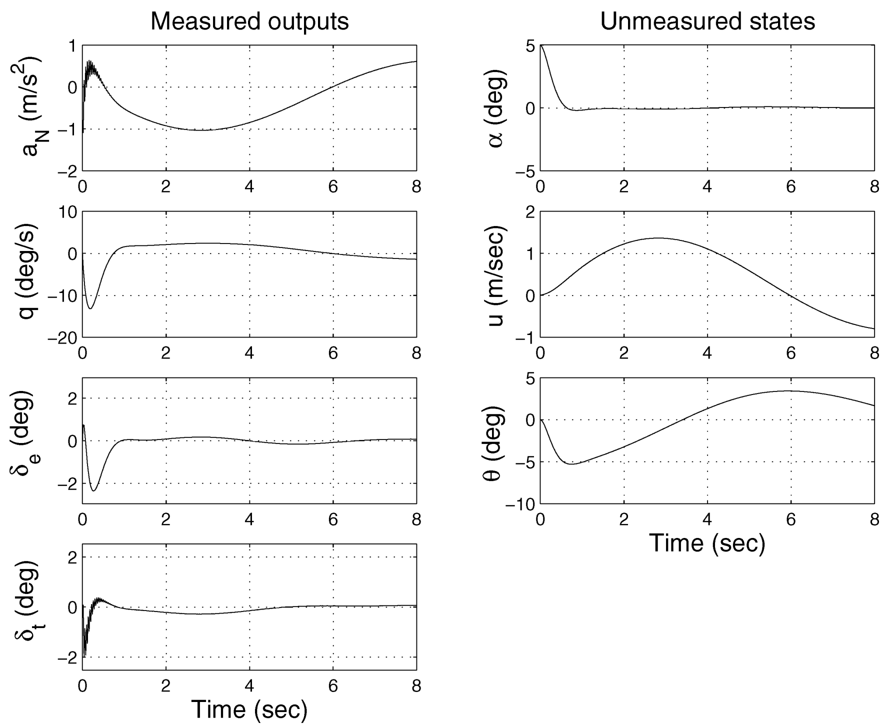

Fig. 1 Closed-loop response for initial $\alpha = 5$ deg.

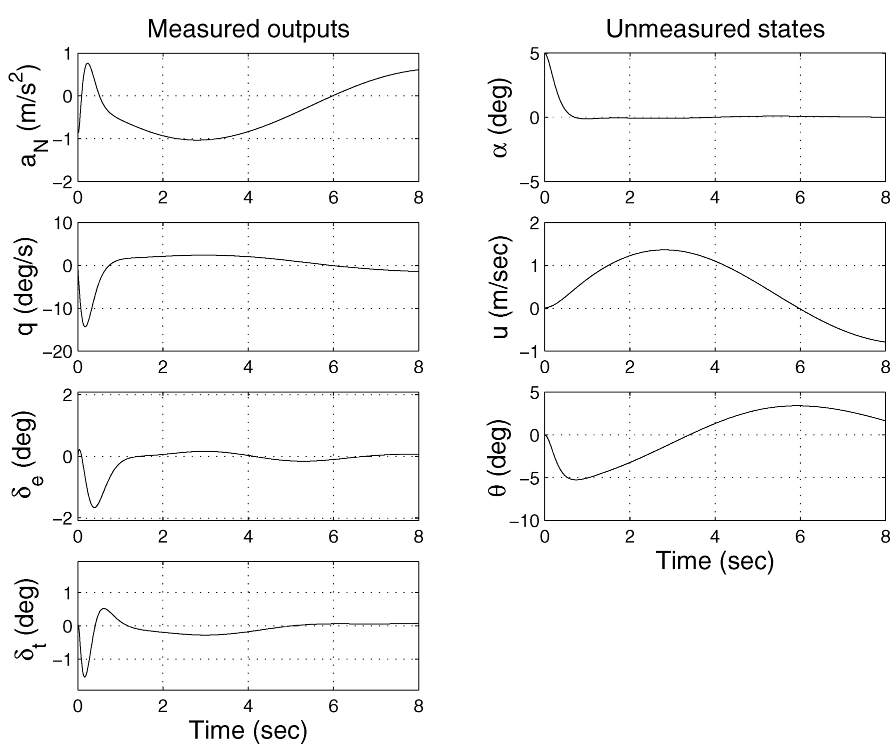

Fig. 2 Closed-loop response for initial $\alpha = 5$ deg and actuator time constants of 250 ms.

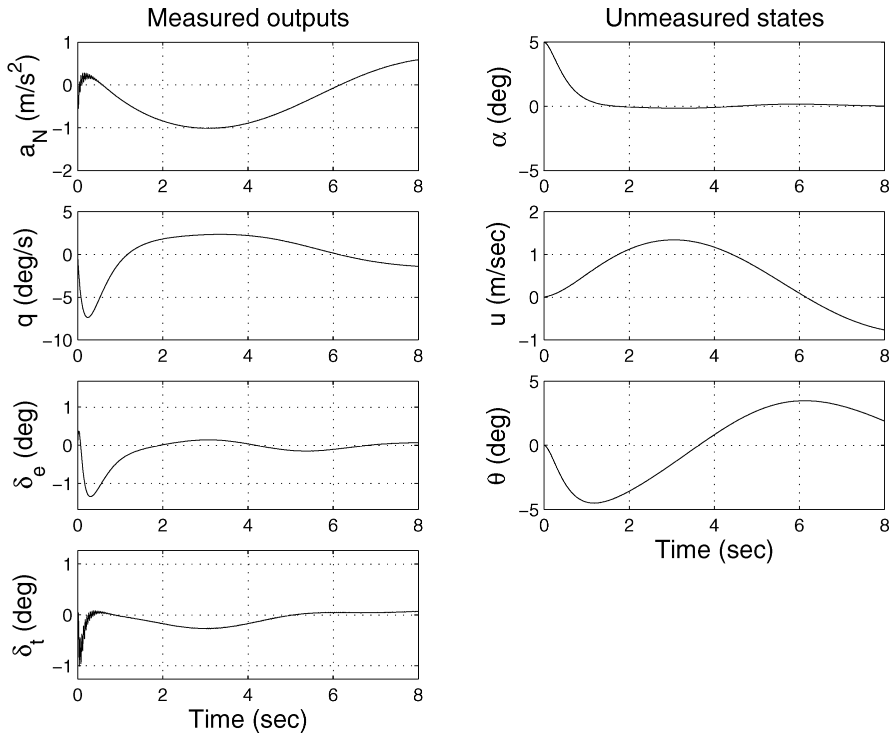

Fig. 3 Closed-loop response for initial $\alpha = 5$ deg and Z_α and M_α at 50% nominal.

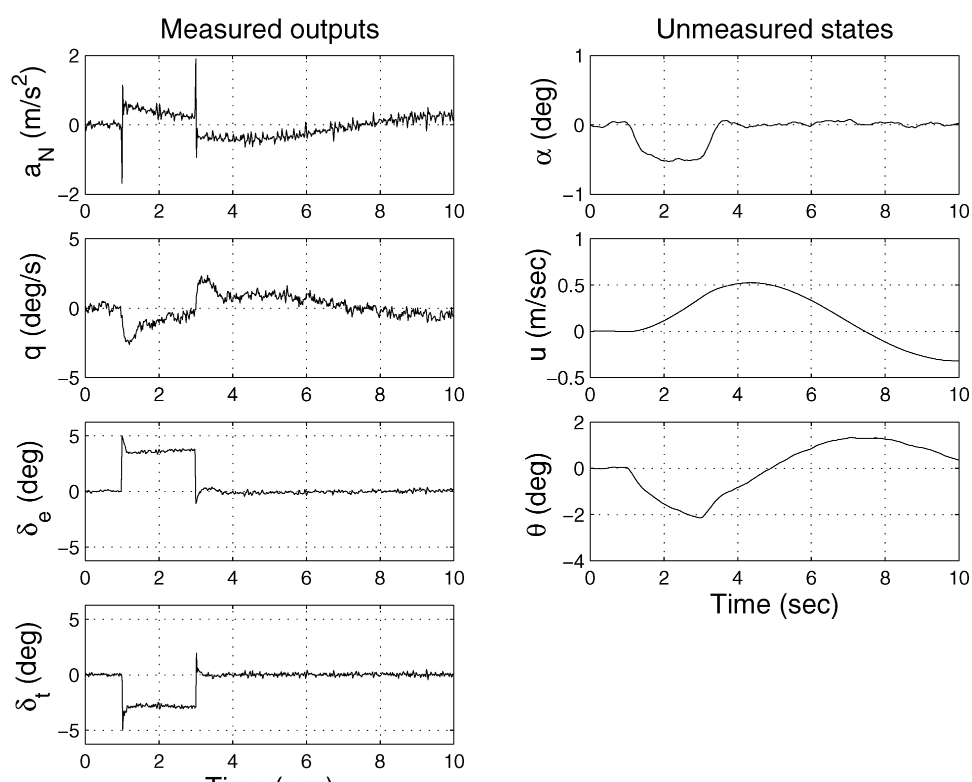

Fig. 4 Closed-loop response with noisy measurements for a disturbance of 5 deg to elevator.

small changes from equilibrium when compared to the open-loop response. Simulation is carried out with actuator time constants changed to five times actual (Fig. 2) and with Z_α and M_α at 50% of nominal (Fig. 3). To verify disturbance-rejection properties, we simulate the closed-loop system with a disturbance input of 5 deg to the elevator (Fig. 4). The responses show that the control system is invariant to large parameter uncertainties in the plant model and exhibits good rejection of disturbance inputs. Control actuation is within acceptable limits set by actuator specifications. It is noted that there has not been any improvement in damping of the low-frequency phugoid response, which is due to the fact that there is no information on the states u and θ that govern this mode. However, amplitudes of oscillations of these states are low in closed-loop operation.

The algorithm developed may lead to a suboptimal controller because the parameter matrices S_p, S_c, P, and H are user-chosen.

There is scope for improvement by applying optimality conditions on the controller performance to compute these matrices.

V. Conclusions

In this Note, a sliding-mode controller is designed with output feedback and dynamic compensation using a novel sliding function. The control law is derived for discrete-time implementation using the equivalent control approach for existence of a quasi-sliding mode. Transformation of the plant model to regular form is not required and new conditions have been imposed on the system matrices. The design procedure involves formulation of the augmented reduced-order system in general output feedback form, allowing application of multivariable feedback design techniques to find parameter matrices of the compensator and the feedback gains. The matrices characterizing the switching surfaces are user-chosen and there are no numerical difficulties with invertibility conditions. The algorithm is applied to design a pitch controller for an aircraft having controllable elevator and thrust deflections and the results of simulation are presented.

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